

Name: \_\_\_\_\_

PC: Partial Fractions

Date: \_\_\_\_\_

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Do Now:

1. Using matrices find the area of the triangle with vertices  $(0,4)$ ,  $(2,3)$ , and  $(5,0)$ .
2. Are the points  $\left(2, \frac{1}{2}\right)$ ,  $(-4,4)$ , and  $(6,-3)$  collinear?
3. Using matrices, find an equation of the line that passes through  $(10,7)$  and  $(-2,-7)$ .

To write a sum or difference of fractional expressions as a single fraction, we find a common denominator. For example:

$$\frac{1}{x-1} + \frac{1}{2x+1} =$$

However, for some applications of algebra to calculus, we must reverse this process. We express a fraction as a sum of simpler fractions. These simpler fractions are called **partial fractions**.

1. Find the partial fraction decomposition of  $\frac{x+6}{x(x+3)}$

2. Find the partial fraction decomposition of  $\frac{2x}{(x-1)(x+1)}$

3. Find the partial fraction decomposition of  $\frac{2}{x^2-1}$

4. Find the partial fraction decomposition of  $\frac{5}{x^2 + 3x - 4}$

Steps:

1. Make sure that the degree of the numerator is less than the degree of the denominator.
2. Express the denominator in completely factored form.
3. Set up the partial fraction decomposition equation by using each of the linear factors of the denominator as the denominators of your partial fractions. Use A, B, C, ..., to represent the constants in the numerators.
4. Multiply the equation by the LCD and simplify the right side by distributing and combining like terms.
5. Form a system of equations by setting the appropriate coefficients on the right side equal to their matches on the left.
6. Solve the system and use your findings to write the partial fraction decomposition.

5. Find the partial fraction decomposition of  $\frac{x-12}{x^2 - 4x}$

6. Find the partial fraction decomposition of  $\frac{5x+7}{x^3+2x^2-x-2}$

7. Find the partial fraction decomposition of  $\frac{2x^3+3x^2-x+16}{x^2+2x-3}$

**Notice here that the degree of the numerator is GREATER than the degree of the denominator. We have to tweak the process.**

8. Find the partial fraction decomposition of  $\frac{x^3 - x + 2}{x^2 - 1}$

**Other Special Cases**

9. Find the partial fraction decomposition of  $\frac{2x - 3}{x^2 - 2x + 1}$

10. Find the partial fraction decomposition of  $\frac{3x}{(x - 3)^2}$

11. Find the partial fraction decomposition of  $\frac{x^2 + 4x + 4}{x^3 - x^2 + 2x - 2}$

12. Find the partial fraction decomposition of  $\frac{2x^2 - x + 4}{x^3 + 4x}$

***Practice Exercises***

1. Find the partial fraction decomposition of  $\frac{2x+1}{x^2+x-2}$
2. Find the partial fraction decomposition of  $\frac{x+14}{x^2-2x-8}$
3. Find the partial fraction decomposition of  $\frac{8x-3}{2x^2-x}$
4. Find the partial fraction decomposition of  $\frac{x}{8x^2-10x+3}$
5. Find the partial fraction decomposition of  $\frac{7x-3}{x^3+2x^2-3x}$
6. Find the partial fraction decomposition of  $\frac{9x^2-9x+6}{2x^3-x^2-8x+4}$
7. Find the partial fraction decomposition of  $\frac{7x+17}{(x+3)^2}$
8. Find the partial fraction decomposition of  $\frac{2x^2-x+4}{x^3+4x}$
9. Find the partial fraction decomposition of  $\frac{2x}{(x-1)(x^2+1)}$
10. Find the partial fraction decomposition of  $\frac{x^2+1}{x^3+2x^2-x-2}$

11. Find the partial fraction decomposition of  $\frac{6+26x-x^2}{(2x-1)(x+2)^2}$

12. Find the partial fraction decomposition of  $\frac{4x^2-14x+2}{4x^2-1}$

13. Find the partial fraction decomposition of  $\frac{12x^4-12x^3+7x^2-2x-3}{4x^2-4x+1}$

14. Find the partial fraction decomposition of  $\frac{2x^4+4x^3-2x^2+x+7}{x^3+2x^2-x-2}$